THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, April 30, extending through the usual morning and afternoon sessions. The following forty-nine members were in attendance:

Mr. Joseph Allen, Dr. Grace Andrews, Professor Maxime Bôcher, Professor C. L. Bouton, Professor Joseph Bowden, Professor E. W. Brown, Professor F. N. Cole, Miss E. B. Cowley, Miss L. D. Cummings, Dr. W. S. Dennett, Dr. L. P. Eisenhart, Dr. William Findlay, Professor T. S. Fiske, Mr. C. S. Forbes, Dr. A. S. Gale, Mr. Clifford Gray, Mr. G. H. Hallett, Mr. E. A. Hook, Dr. E. V. Huntington, Mr. L. L. Jackson, Mr. S. A. Joffe, Dr. Edward Kasner, Dr. O. D. Kellogg, Professor C. J. Keyser, Dr. G. H. Ling, Mr. L. L. Locke, Professor E. O. Lovett, Professor James Maclay, Professor H. P. Manning, Professor W. H. Metzler, Professor E. H. Moore, Dr. L. I. Neikirk, Professor James Pierpont, Dr. I. E. Rabinovitch, Miss Virginia Ragsdale, Mr. F. G. Reynolds, Dr. Arthur Schultze, Professor Charlotte A. Scott, Professor D. E. Smith, Professor Virgil Snyder, Professor Henry Taber, Professor J. H. Tanner, Professor H. D. Thompson, Professor H. W. Tyler, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Dr. E. B. Wilson, Dr. Ruth G. Wood, Professor R. S. Woodward.

The President of the Society, Professor Thomas S. Fiske, occupied the chair, being relieved by Vice-President Professor J. M. Van Vleck. The Council announced the election of the following persons to membership in the Society: Mr. J. J. Brown, Colorado School of Mines; Mr. C. E. Dimick, University of Pennsylvania; Dr. William Gillespie, Princeton University; Mr. Clifford Gray, Columbia University; Mr. Louis Ingold, University of Missouri; Professor Myrtle Knepper, State Normal School, Cape Girardeau, Mo.; Professor F. M. Morrison, Buchtel College; Professor G. W. Myers, University of Chicago; Mr. Elijah Swift, Harvard University. Eight applications for admission to the Society were received.

For the days preceding the meeting, sessions of various committees, including the editorial staff of the *Transactions*, were arranged. The committee appointed at the annual meeting of 1902 to consider the relation of the Society to elementary mathematics presented a final report reciting the organization, under the Society's influence, of several active associations of teachers of mathematics. The committee was discharged at its own request.

It was decided to hold the summer meeting of the Society at St. Louis on Friday and Saturday, September 16–17.

The following papers were read at the meeting:

(1) Dr. H. F. STECKER: "Certain differential equations in relation to non-euclidean geometry."

(2) Professor C. J. KEYSER: "Certain line and plane quintic configurations in 4-space, and their sphere analogues in ordinary space."

(3) Dr. E. V. HUNTINGTON: "Sets of independent postulates for the algebra of logic (third paper)."

(4) Dr. O. D. KELLOGG: "Functions with preassigned singular points and monodromic group."

(5) Professor J. M. PEIRCE: "On certain complete systems of quaternion expressions, and on the removal of metric limitations from the calculus of quaternions."

(6) Professor E. O. LOVETT: "Singular trajectories in the problem of four bodies."

(7) Professor E. O. LOVETT: "Systems of periplegmatic orbits."

(8) Professor MAXIME BÔCHER: "The Gauss-Stieltjes equilibrium problem, and the roots of polynomials."

(9) Professor JAMES PIERPONT: "On multiple integrals."

(10) Mr. E. L. DODD: "Multiple sequences."

(11) Professor VIRGIL SNYDER: "On developable and tubular surfaces having spherical lines of curvature."

(12) Mr. C. H. SISAM: "On self-dual scrolls."

(13) Dr. EDWARD KASNER: "The general transformation theory of differential elements."

(14) Professor E. J. WILCZYNSKI: "General theory of curves on ruled surfaces."

(15) Mr. R. B. ALLEN: "On real hypercomplex number systems."

Mr. Dodd was introduced by Professor Pierpont. Mr. Sisam's paper was presented to the Society through Professor

Snyder, and Mr. Allen's through Professor Taber. In the absence of the authors, Professor Peirce's paper was read by Professor Bôcher, Mr. Sisam's by Professor Snyder, Mr. Allen's by Professor Taber, and the papers of Dr. Stecker and Professor Wilczynski were read by title. Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Starting from the relation $ds = 4\sqrt{dx^2 + dy^2}/\Theta$, where $\Theta \equiv k_1 y_1 \bar{y}_1 + k_2 y_2 \bar{y}_2$ and in which y_1 and y_2 are a pair of independent solutions of a linear homogeneous differential equation of determinant unity, and recalling that the curvature reduces to $K = k_1 k_2$, Dr. Stecker obtains interesting relations between differential equations of this type and non-euclidean geometry. If $\Theta^2 e^u = 4$, then $\Delta u = -2k_1 k_2 e^u$, and because of the above two relations it becomes possible to express the non-euclidean trigonometric functions in terms of the solutions of this last equation $\Delta u = -2k_1 k_2 e^u$.

2. Let the terms range, congruence, complex, of spheres, signify respectively the one-, two-, three-dimensional sphere assemblages of which respectively the elements are common to a circle, orthogonal to a circle, orthogonal to a sphere. Suppose established a one-one correspondence: (1) in 4-space, between points and lineoids (ordinary 3-spaces, linear point complexes), and therewith between point ranges (and hence also their bases, lines) and point congruences (and their bases, planes); (2) in 3-space, between spheres and sphere complexes, and therewith between sphere ranges (and hence also their bases, circles) and sphere congruences; (3) between 3-space spheres and 4-space points, and therewith between 3-space sphere ranges (circles) and 4-space point ranges (lines) and between 3-space sphere congruences and 4-space point congruences (planes). This done, the 3-space reciprocal 4-dimensional sphere and sphere complex theories then stand in a fact to fact relation with the point and lineoid theories of 4-space; and the 3-space reciprocal 6-dimensional sphere range (circle) and sphere congruence theories stand respectively in the like relation with the 4-space point range (line) and point congruence (plane) theories. The four mentioned 3-space geometries, while logically equivalent, are psychologically distinctly more difficult than their 4-space correlates, owing to the apparent

relative simplicity of the elements employed in the latter. It is accordingly no trivial principle of economy to construct these theories first, and then the former by the process of translation or exchange of notions. A chief aim of Professor Keyser's paper is to illustrate the operation and advantage of this principle. Among the theorems established and used for that purpose may be stated the following correlates: Three independent lines of 4-space have one and but one common intersector; one and but one plane is collineoidal with each of three given independent planes, or intersects each of them in a line; one and but one sphere range (circle) has a sphere in common with each of three given independent sphere ranges (circles); one and but one sphere congruence lies with each of three given independent sphere congruences in a sphere complex, or intersects each of them in a sphere range. (The *circle* aspect of the next to the last proposition was presented by Darboux in *Comptes rendus*, volume 92). Again, four independent point ranges (lines), or sphere congruences (planes), or sphere ranges (circles), or sphere congruences, determine uniquely a covariant fifth element of the same kind, the quintic configuration so arising being such that any four elements of it determine the fifth. The method of constructing the latter from the former is given and leads to other more complicated configurations of remarkable symmetry. The quintic group of "circles" was otherwise found in 1881 by Cyparissos Stephanos.

3. Dr. Huntington's first set of postulates for the algebra of logic, presented at the summer meeting, 1903, was based on two "rules of combination," $\oplus$ and $\odot$, interpretable as "logical addition" and "logical multiplication." The second set, presented in December, 1903, was based on a dyadic relation, $\circledcirc$, interpretable as "inclusion" or "subsumption." This set has since been revised in view of a demonstration of the distributive law communicated to the writer by Mr. C. S. Peirce. The third set, proposed in the present paper, is based on a single rule of combination, "addition." An appendix contains a simple rule for constructing any finite *logical field* (that is, any finite system which obeys the laws of the algebra of logic), the number of elements being a power of 2. All the papers have been offered together to the *Transactions*.

4. The existence of a set of n analytic functions of a complex variable z with m preassigned singular points such that when z

completes a circuit about each of them the n functions go over into linear functions of themselves with preassigned constant coefficients has been established by Schlesinger for the case that the roots of the fundamental equations of the linear substitutions belonging to each singular point have moduli equal to unity. It was Hilbert's suggestion in a course of lectures (Potentialtheorie, 1901–1902,) to regard this as a boundary problem. The functions are then sought as the logarithmic potential of a distribution of matter along a curve, and the distribution moment will be determined as the solution of a functional equation of the Fredholm type, which however contains discontinuous functions in such a way as to make the immediate application of his method impossible. These discontinuities can, however, be eliminated, so that it is possible to assert the existence of at least one set of functions with the required properties.

5. Professor Peirce's paper has a twofold object. First, it proposes certain systems (to be called *complete*) of reciprocally related forms which may often be usefully employed, and of which Hamilton furnishes a simple example in his discussion of the theory of the linear vector function, and in other places in the Elements. Secondly, it seeks to remove the metric limitations which appear to belong to the calculus of quaternions in its usual elementary presentation, and to show that these limitations are not inherent in the calculus itself. For this purpose, planes as well as points are denoted by vectors, the vector of a plane being the same as that of the pole of the plane, relatively first to a unit sphere, then to any quadric. Thus, the principle of dualism between points and planes is brought into quaternions. A further step is to represent points and planes alike by quaternions instead of vectors. Thus infinites are avoided, and the full freedom of a projective system is attained. By the aid of "complete systems" of vectors and quaternions, the plan here proposed is fully shown to be the quaternion equivalent of the method of tetrahedral coördinates.

6. In a recent memoir inserted in the ninth volume of the *Annali di Matematica*, Professor Levi-Civita has studied the conditions of collision in the restricted problem of three bodies and the irregular trajectories on which collisions are possible. Professor Lovett's note extends the results of Levi-Civita's

investigations to the restricted problem of four bodies in which one is infinitesimal, the other three being finite and in motion according to one of the exact solutions of the problem of three bodies. Painlevé has shown that in the problem of n bodies the conditions of collision are certainly transcendental if three or more of the masses are different from zero. Levi-Civita finds in the memoir cited that these transcendental conditions become again algebraic in the restricted problem of three bodies; this furnishes no exception to Painlevé's theorem since but two of the masses are different from zero in the restricted problem of three bodies. Levi-Civita's analysis can be made immediately available for the restricted problem of four bodies in which the three finite bodies, all of mass different from zero, are in the equilateral triangle solution of Lagrange; this case of algebraic singular trajectories constitutes a curious exception to Painlevé's theorem just quoted. The note will appear in the eleventh volume of the *Annali di Matematica*.

7. The paper on Gyldén's periplegmatic orbits * by Professor Lovett was inspired by Dr. G. W. Hill's memoir † on pairs of such orbits which appeared in the *Astronomical Journal*. The successive sections of the note are occupied with triple and n-ple systems of plane periplegmatic orbits. There appears at the end a postscriptum which has to do with certain pairs of entangled plane orbits, periplegmatic or otherwise, whose determination depends either on elliptic functions or those new uniform transcendental functions recently discovered by Painlevé.‡ The method of discussion employed is essentially that used by Hill, and the generalizations constructed are suggested very naturally by the examples of his memoir. The note was published in the *Astronomical Journal*, May 2, 1904.

8. The starting point in Professor Bôcher's paper was the problem of determining the field of force in a plane due to a number of fixed particles in the plane which repel inversely as the distance and directly as the masses. In particular the points

* H. Gyldén, Traité analytique des orbites absolues des huit planètes principales, vol. I, livre I, chap. I.

† G. W. Hill, "Examples of periplegmatic orbits," *The Astronomical Journal*, vol. 24, No. 2, pp. 9–14 (January 21, 1904).

‡ P. Painlevé, "Sur les équations différentielles du second ordre et d'ordre supérieur dont l'intégrale générale est uniforme," *Acta Mathematica*, vol. 25, pp. 1–86.

in the plane where a moveable particle can rest in equilibrium are sought (cf. for a special case Gauss, Collected Works, volume 3, page 112). The problem will not be essentially changed if the particles are situated not in a plane but on a spherical surface, provided the sum of the masses of the particles is then zero. This being assumed to be the case, and the sphere being taken for the sphere of complex numbers, connection is easily made, by introducing homogeneous variables, with the theory of binary forms; and it is found that if the positions of the particles are determined by equating the ground forms to zero (only particles of equal mass being grouped together so as to correspond to the same ground form), the positions of equilibrium are determined by the roots of a certain covariant — in the case of two ground forms their jacobian. Certain theorems concerning the position of these covariant points are immediately suggested by the mechanical problem.— All these considerations may be generalized by seeking the positions of equilibrium not of a single particle but of a group of unit particles which are not only acted on by the fixed field of force so far considered but also repel one another. This is the problem first considered by Stieltjes, though only for the case of one dimension, and leads to Stieltjes' polynomials as solutions of certain linear homogeneous differential equations of the second order which are everywhere regular. This subject also is brought in the present paper into connection with the theory of algebraic invariants. The problem may, however, be varied by introducing other fields of force. Thus, for instance, a simple mechanical problem leads to Hermite's polynomials which, it may be noted, differ only by a very simple exponential factor from certain special cases of the functions of the parabolic cylinder.

9. To deduce the usual properties of a multiple integral over a point aggregate $\mathfrak{A}$ in n dimensions, it has been found necessary heretofore to assume that $\mathfrak{A}$ is *measurable*. In Professor Pierpont's paper it is shown that this restriction is unnecessary. We may in fact enlarge our definition of a multiple integral in such a manner that the old and new definitions become identical when $\mathfrak{A}$ is measurable. As a result of the new definition not only is the theory of the integrals generalized but several fundamental theorems may be demonstrated with extreme simplicity which in the older theory required lengthy consideration.

10. Pringsheim and London have given extended treatments of double sequences and Bromwich has also obtained some theorems regarding these sequences as a preface to his discussion of double integrals. In Mr. Dodd's paper these results have been generalized for multiple sequences of any order. The generalization is not always obvious, and the case of multiple sequences of order greater than two has given rise to a number of new theorems.

11. In Professor Snyder's paper various known theorems regarding the lines of curvature on annular surfaces were proved in a very elementary manner by means of spherical geometry, without the use of differential equations.

12. Mr. Sisam proved the following theorem by algebraic processes: The necessary and sufficient condition that a scroll is self-dual is that it belongs to a non-special linear complex. The paper appeared in the June number of the BULLETIN.

13. The simplest species of element transformation is that which converts every lineal element (x, y, y') into a lineal element (x_1, y_1, y_1'). Lie's contact transformation is the very special case where every curve (including point) is converted into a curve. With respect to the general case, Dr. Kasner's fundamental result is that there exist a double infinity of curves, the solutions of a certain differential equation of the second order, which alone are converted into curves. It follows that if an element transformation has more than ∞^2 such curves then it is necessarily a contact transformation, so that the ordinary definition of the latter contains redundant requirements.

The paper considers also the general transformation of the elements of higher order $(x, y, y', y'', \cdots)$. In the case of curvature elements, for example, it is shown that while in general no curves are converted into curves, there may exist a finite number or a single, double, or triple infinity of such curves; there cannot be more, except in the case of the contact transformation. In conclusion the extension of the results to space is indicated.

14. In this paper Professor Wilczynski deduces some equations which are of importance in the general theory of curves on ruled surfaces. A number of applications are indicated. The bulk of the paper is devoted to the consideration of ruled

surfaces which have one branch of their flecnode curve or of their complex curve arbitrarily assigned. It is shown that the general expression of these surfaces contains an arbitrary function, and a geometrical construction for the surfaces is given. A very remarkable theorem appears, demonstrating the existence of a family of ∞^1 surfaces, having one of the branches of their flecnode curves in common, and forming an involution. This theorem is as follows:

1. If at every point of the flecnode curve of S there be drawn the generator of the surface, the flecnode tangent, the tangent to the flecnode curve, and finally the line which is the harmonic conjugate of the latter with respect to the other two, the locus of these last lines is a developable surface, the secondary developable of the flecnode curve.

2. One can find a single infinity of ruled surfaces, each having one branch of its flecnode curve in common with that of S. This family of ∞^1 surfaces can be described as an involution of which any surface of the family and its flecnode surface form a pair. The primary and secondary developables of the branch of the flecnode curve considered are the double surfaces of this involution. In fact, the generators of these surfaces, at every point of their common flecnode curve, form an involution in the usual sense.

15. Mr. Allen's paper is in abstract as follows: The units of a real non-nilpotent hypercomplex number system may be so chosen as to accord with Peirce's normal form, that is the system may be regularized with respect to an idempotent number e_0. The units of the first group, other than e_0 may then be so selected as to fall into one or the other of two classes: the first consisting of nilpotent numbers $e_1, e_2, \cdots, e_\mu$, forming a system by themselves, and as many in number as there are nilpotent numbers in the first group; the second class consisting of units $e_{\mu+1}, e_{\mu+2}, \cdots, e_{\mu+\nu}$, satisfying the condition $e_h^2 = -e_0$ $(h = \mu+1, \mu+2, \cdots, \mu+\nu)$, and forming with e_0 a system by themselves, when so chosen we have $\mu = m(\nu+1)$, and $\nu = 0$, 1, or 3. When $\nu = 3$, the units $e_{\mu+1}, e_{\mu+2}, e_{\mu+3}$ of the second class may be selected so that $e_{\mu+1}, e_{\mu+2} = e_{\mu+3}$; and thus in this case $e_0, e_{\mu+1}, e_{\mu+2}, e_{\mu+3}$ constitute the Hamiltonian quaternion system.

F. N. Cole.

Columbia University.

THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE fifth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at Stanford University, on Saturday, April 30, 1904. The following members were present:

Professor R. E. Allardice, Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Dr. D. N. Lehmer, Mr. W. A. Manning, Professor G. A. Miller, Professor H. C. Moreno, Professor C. A. Noble, Professor Irving Stringham.

The attendance also included a number of teachers of mathematics who are not members of the Society. A morning and an afternoon session were held, Professor Allardice acting as chairman at both sessions. It was decided to hold the next meeting at the University of California on October 1, 1904.

The following papers were read at this meeting:

(1) Professor M. W. HASKELL: "The construction of conics satisfying given conditions."

(2) Professor H. C. MORENO: "On a class of ruled loci."

(3) Dr. D. N. LEHMER, "On a cylinder the intersection of which with a sphere will develop into an ellipse."

(4) Professor R. E. ALLARDICE: "On the envelope of the directrices of a system of similar conics through three points."

(5) Mr. A. W. WHITNEY: "The application of actuarial methods to fire insurance."

(6) Professor IRVING STRINGHAM: "Analytical treatment of certain metric relations in the non-euclidean plane."

(7) Professor G. A. MILLER: "Addition to a theorem due to Frobenius."

(8) Professor H. F. BLICHFELDT: "On the primitive collineation groups in four variables."

(9) Professor H. F. BLICHFELDT: "A theorem concerning the invariants of linear homogeneous groups, with some applications to substitution groups."

(10) Professor M. W. HASKELL: "The construction of a twisted cubic from six points."

(11) Professor M. W. HASKELL: "Triangles in perspective and the collineations derived therefrom."

Mr. Whitney's paper was read by Professor Stringham. All the other papers were read by their authors. Abstracts are given below. The abstracts are numbered to correspond to the titles in the list above.

1. The main feature of Professor Haskell's paper is the relation between the various problems in the determination of conics and the involutory quadratic transformation. The author finds that the points of contact of the two conics passing through four given points and tangent to a given line are converted into each other by the involutory quadratic transformation of which the four given points are the self-corresponding points, the diagonal triangle of the complete quadrangle of these four points being the fundamental triangle of the transformation.

There are four conics passing through two points and tangent to three given lines, and they will be fully determined if the poles of their common chord are given. It is shown that these poles are the self-corresponding points of the involutory quadratic transformation interchanging the two given points, the fundamental triangle being the triangle of the three given tangents.

2. Professor Moreno's paper is devoted to an extension of a theorem due to Salmon (Geometry of three dimensions, 1882, page 429). The problem as extended is to find the locus of all the lines that meet n $(n-2)$-spreads in space. The simplest and most interesting case presents itself when the director spreads are flat. In this case the degree of the locus is $n-1$; $(n-1)!$ different generators go through any one point on this locus and these generators belong to different systems. There are, in general, $n(n-1)$ $(n-2)$-flats on this locus. Exceptions occur for $n=3$. The locus when $n=4$ has been studied by Segre. In case the director spreads are of orders $m_1, m_2, \cdots, m_n$ respectively, the orders of the loci are, in general, $(n-1)m_1 m_2 \cdots m_n$, and the spread of order m_1 is generally of multiplicity $m_2 m_3 \cdots m_n$.

3. In his Mathematical Recreations published in 1624 Van Etten (Jean Leurechon) describes a method of drawing an

"oval" with a pair of compasses, leaving the angle of the instrument unaltered. His scheme is to stretch the drawing paper around a circular cylinder. The resulting curve is not an ellipse, the equation being $y^2 + 4b^2 \sin^2(x/2b) = a^2$, where b is the radius of the cylinder and a the spread of the compasses. The curve is clearly the development of the intersection of a sphere with a cylinder, the center of the sphere being on the cylinder.

The question suggests itself to find the cylinder such that its curve of intersection with a sphere, the center of which is on the cylinder, will develop into an ellipse. Dr. Lehmer shows that a right section of the cylinder is a logarithmic spiral. The solution indicates that if an elliptical hole be made in a sheet of paper and the sheet wrapped about a sphere the radius of which equals the minor axis of the ellipse, the sheet wraps up into a cylinder whose right section is a logarithmic spiral.

4. Mr. Whitney's paper relates to the construction of fire insurance tables showing the premiums for various ratios of insurance to value. The derivation of such tables for several classes of risks has been published in the *Proceedings* of the twenty-eighth annual meeting of the Fire underwriters' association of the Pacific.

5. In this paper Professor Allardice continues the discussion of a series of problems, suggested by Steiner. An explicit equation for the envelope of the directrices is obtained in a comparatively simple form. The envelope is a curve of the fourth class with one double tangent, the straight line at infinity, the circular points being the points of contact. The centres of the inscribed circle and the three escribed circles of the triangle formed by the fixed points are double points on the envelope, for every value of the eccentricity. In a paper in the *Bulletin des sciences mathématiques* the envelope has been stated by Schoute to be a curve of the eighth class.

6. Professor Stringham's paper sets up the purely analytical criteria for determining the characteristics of the fundamental types of the non-euclidean geometries. The coördinates of points and of straight lines being w, x, y and ω, ξ, η respectively, the assumed fundamental relations are $w^2 + x^2 + y^2 = 1$, $\omega^2 + \xi^2 + \eta^2 = 1$, and the distance functions are the following:

Between points and points $f = ww' + xx' + yy'$, between lines and lines (angle) $c = \omega\omega' + \xi\xi' + \eta\eta'$, between points and lines $g = k(\omega' w + \xi' x + \eta' y)$, where k is the fundamental space constant.

If the points (w), (w') lie upon the lines (ω), (ω') respectively, it is shown that maxima and minima of f exist and that the conditions for these maxima and minima make the points (w), (w') lie on the absolute polar of the intersection of the lines (ω), (ω'). This result makes it possible to replace the distance function f by g and to express g as a function of w alone, and the criteria for the maxima and minima of g are then easily determined. They show that g has maxima when k^2 is positive, minima when k^2 is negative, provided $\xi y - \eta x$ and $\xi' y' - \eta' x'$ be not infinite. The well known result follows that any two distinct straight lines in the elliptic plane (k^2 positive) have a maximum distance apart measured along the absolute polar of their real point of intersection, and that any two non-intersectors in the hyperbolic plane (k^2 negative) have a minimum distance apart measured along the absolute polar of their imaginary point of intersection.

7. In a paper recently presented before the London mathematical society, Professor Miller proved that the number of cyclic groups of order p^α ($p > 2$, $\alpha > 1$) in any group G whose Sylow subgroups of order p^m are non-cyclic is always a multiple of p. From this it follows directly that a non-cyclic group of order p^m contains $p^\alpha k$ operators whose order is p^α and hence the number of its operators which satisfy the equation $s^{p^\alpha} = 1$ is a multiple of $p^{\alpha+1}$. The theorem of Frobenius states that this number is a multiple of p^α. In any group G whose Sylow subgroups of order p^m are non-cyclic the number of operators of order $p^\alpha m$, m being prime to p, is a multiple of p^α.

8. Professor Blichfeldt's paper is devoted to an enumeration of the primitive collineation groups in four variables.

9. The results given in the second paper by Professor Blichfeldt are as follows: Let $\theta_1, \theta_2, \cdots, \theta_n$ be the multipliers of a substitution S_i of a linear homogeneous group G of order N in n variables. If the sum of the homogeneous products of degree m be formed from $\theta_1, \theta_2, \cdots, \theta_n$ and be designated by W_p then

$$N^{-1}\sum_{i=1}^{N} W_i$$

is equal to the number of linearly independent absolute invariants of degree m of the group G. Also, let w_i be any symmetric function of the multipliers $\theta_1, \cdots, \theta_n$ then the same sum is an integer (positive or negative) or zero. Applying this principle to substitution groups, a number of known theorems concerning such groups are derived.

10. Professor Haskell's second paper is in abstract as follows: Let $A_1, A_2, A_3, A_4, A_5, A_6$ be the given points, and let it be required to find the third point A_7 which lies on any plane through A_5 and A_6. The lines joining the first four points in pairs meet this plane in the six vertices of a complete quadrilateral. Denoting by A_{ik} the intersection of this plane with the line joining A_i and A_k, then A_{12} and A_{34}, A_{13} and A_{24}, A_{14} and A_{23} are interchanged by the involutory quadratic transformation of which $A_5\ A_6\ A_7$ is the fundamental triangle.

11. After determining the condition that two triangles in perspective should be interchanged by a collineation of period 2, Professor Haskell shows that any collineation which leaves a conic invariant can be reduced to the product of two perspective collineations in an infinity of ways and that any collineation whatever is equivalent to the product of four perspective collineations. Then follow the deduction and geometric interpretations of the well-known groups of collineations in the plane.

G. A. MILLER,
Secretary of the Section.

ON LINEAR HOMOGENEOUS DIFFERENCE EQUATIONS AND CONTINUOUS GROUPS.

BY DR. SAUL EPSTEEN.

(Read before the Chicago Section of the American Mathematical Society, April 2, 1904, under the title, "On Linear Homogeneous Difference Equations.")

IN the *Comptes Rendus* of October 27, 1903, there appeared a paper of fundamental importance by Dr. Alfred Guldberg, of Christiania, Norway, entitled "Sur les groupes de transformations des équations linéaires aux différences finies." Section 1 will be a brief summary of Guldberg's theory, the remainder of this note being devoted to the questions of numerical invariance, domain of rationality and algebraic integrability, which are not considered there.

1. Let $[y_x] \equiv y_x^{(1)}, y_x^{(2)}, \cdots, y_x^{(n)}$ be a fundamental system of solutions of the linear homogeneous equations in finite differences

$$y_{x+n} + p_x^{(1)} y_{x+n-1} + \cdots + p_x^{(n)} y_x = 0 \tag{1}$$

and let

$$V_x = \alpha_x^{(1)} y_x^{(1)} + \cdots + \alpha_x^{(n)} y_x^{(n)} \tag{2}$$

be a linear function with rational coefficients of these solutions, which may be known as the Guldberg resolvent of (1). This resolvent satisfies an equation of order n^2

$$V_{x+n^2} + P_x^{(1)} V_{x+n^2-1} + \cdots + P_x^{(n^2)} V_x = 0. \tag{3}$$

From (2) it is seen that

$$\begin{aligned} y_x^{(1)} &= \alpha_x^{(1)} V_x + \alpha_x^{(2)} V_{x+1} + \cdots + \alpha_x^{(n^2)} V_{x+n^2-1}, \\ y_x^{(2)} &= \beta_x^{(1)} V_x + \beta_x^{(2)} V_{x+1} + \cdots + \beta_x^{(n^2)} V_{x+n^2-1}, \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ y_x^{(n)} &= \lambda_x^{(1)} V_x + \lambda_x^{(2)} V_{x+1} + \cdots + \lambda_x^{(n^2)} V_{x+n^2-1}, \end{aligned} \tag{4}$$

the $\alpha_x, \beta_x, \cdots, \lambda_x$ being all rational in x.

To each solution of (3) there corresponds a system of solutions $[y_x]$ of (1); this system will be a fundamental one if the determinant of the y_x and of their successive values up to the order $n-1$ does not vanish; this determinant being written in terms of V_x gives an equation of the form

$$\phi(x, V_x, V_{x+1}, \cdots, V_{x+k}) = 0, \tag{5}$$

k being at most equal to $n^2 - 1$. Let

$$f(x, V_x, V_{x+1}, \cdots, V_{x+p}) = 0 \tag{6}$$

be the irreducible algebraic equation of lowest order which is satisfied by the solution V_x of (3) which is not at the same time a solution of (5); let $y_x^{(1)}, \cdots, y_x^{(n)}$ be the fundamental system of (1) corresponding to V_x and let $z_x^{(1)}, \cdots, z_x^{(n)}$ be the system corresponding to the general solution of (6); we have

$$\Gamma: \qquad z_x^{(i)} = \sum_{j=1}^{n} a_{ij} y_x^{(j)} \qquad (i = 1, \cdots, n), \tag{7}$$

the a_{ij} being algebraic functions of p arbitrary parameters. The totality of these substitutions, which evidently form a group, is called *the group* Γ *of the equation* (1).

Guldberg demonstrates the following theorems which are entirely analogous to certain theorems in the Galois theory of algebraic equations and in the Picard-Vessiot theory of linear homogeneous differential equations:*

A. *Every rational function of* $x, y_x^{(1)}, \cdots, y_x^{(n)}$ *and their successive values, expressible rationally as a function of* x, *remains invariant when the* $[y_x]$ *are transformed according to the substitutions of* Γ; *every function rational in* x, *the* $[y_x]$ *and their successive values which remains invariant under the substitutions of* Γ *is a rational function of* x.

B. *In order that the equation* (1) *may be integrable by finite quadratures, it is necessary and sufficient that the group* Γ *shall be integrable.*

C. *An equation of order greater than the first cannot in general be integrated by finite quadratures.*

2. In the Galois theory of algebraic equations and in the Picard-Vessiot theory of linear differential equations it is customary to distinguish between the *formal* and *numerical invari-*

* Picard, Traité d'Analyse, vol. 3, chapters 16, 17.

ance of the rational functions of the solutions. It seems to me that this point should be included in an exposition of the Guldberg theory of difference equations and continuous groups. It is well possible for a rational function $\psi(x, [y_x], [y_{x+1}], \cdots) = r(x)$ to remain numerically invariant (*i. e.*, as a function of x) without being formally so.

The same omission was made by Vessiot in his fundamental memoir* on linear differential equations, thus necessitating a revision of his proofs for the case of numerical invariance.† While all of Guldberg's proofs are valid, a similar modification should and can be made. Connected with every linear homogeneous difference equation (1) there are two groups; first, the *Guldberg group* Γ which leaves the totality of rational functions ψ formally invariant, and secondly, a group G which we will call the *group of rationality of* (1) which may alter the form of the ψ's, leaving intact their values as functions of x. The fundamental double theorem A (§ 1), becomes now the following (and, being characteristic, may also be used as definitional):

A'. Every rational function of the elements of a fundamental system of (1) *and of their successive values which is equal to a rational function of x remains numerically invariant (as a function of x) under the transformations of G; and conversely, every rational function of the elements of a fundamental system and of their successive values which is numerically unaltered by the transformations of G is a rational function of x.*

The proof of this double theorem is exactly analogous to the corresponding theorem in the Picard-Vessiot theory.‡

The group G of the difference equation (1) is not uniquely determined, depending upon the selected fundamental system of of solutions $y_x^{(1)}, \cdots, y_x^{(n)}$. Passing from the fundamental system $[y_x]$ to another $[\bar{y}_x]$ which is related to the former by the linear substitution $[y_x] = S[\bar{y}_x]$, there corresponds to the new fundamental system the group of rationality

$$\overline{G} \equiv S^{-1}GS.$$

Thus every subgroup of the general linear homogeneous group which is conjugate with G may be regarded as the group of

* E. Vessiot, *Annales de l'Ecole Normale Supérieure*, 1892.

† Klein, Höhere Geometrie, II, p. 300.

‡ See Schlesinger's Handbuch der Theorie der linearen Differentialgleichungen, II, 1; pp. 71–73.

rationality of (1). When G is invariant in the general linear homogeneous group, the group of rationality becomes independent of the original fundamental system of solutions.

3. In order to obtain the group of rationality G of the equation (1), a *domain of rationality* R should be specified which contains at least the coefficients $p_x^{(1)}, \cdots, p_x^{(n)}$ of the equation. The theorems of § 1, as modified in § 2, remain valid when we *adjoin* to R other known functions $f_1(x), f_2(x), \cdots$, provided that the expression "rational function of x" is replaced by "rational function of $x, f_1(x), f_2(x), \cdots$." It is not necessary that $f_1(x)$, $f_2(x), \cdots$, should appear explicitly in the coefficients p_x of (1).

For the general linear homogeneous equation in finite differences

$$(8) \qquad \begin{vmatrix} y_x & y_x^{(1)} & \cdots & y_x^{(n)} \\ y_{x+1} & y_{x+1}^{(1)} & \cdots & y_{x+1}^{(n)} \\ \vdots & \vdots & & \vdots \\ y_{x+n} & y_{x+n}^{(1)} & \cdots & y_{x+n}^{(n)} \end{vmatrix} = 0,$$

which is satisfied by the indeterminates $y_x^{(1)}, \cdots, y_x^{(n)}$, the coefficients constitute the domain of rationality in which the group of rationality is the general linear homogeneous group.

If a rational function α of the $[y_x]$ and their successive values be adjoined to R, the group of rationality G becomes that subgroup of the general linear homogeneous group which leaves this rational function invariant. In proof, it is clear that G has the necessary double property of § 2, in view of the fact that, the $[y_x]$ being indeterminate, formal and numerical invariance are the same.

When the Guldberg resolvent (3) is such that none of its solutions (which do not satisfy (5)) satisfy an algebraic equation in finite differences of order lower than n^2, the group G will evidently be the general linear homogeneous group itself. In this case the equation (1) with rational coefficients has the same "group character" as (8).

Assume that there exist a certain number of algebraic relations with rational coefficients among the elements of a fundamental system $[y_x]$ (including, if desired, their successive values). We must regard these algebraic relations, in a certain sense, as adjoined to our original domain of rationality and in this way the group of rationality is reduced to the largest subgroup of the general linear homogeneous group which leaves these relations invariant. *The existence of algebraic equations*

with rational coefficients among the $[y_x]$ *and their successive values implies thus certain group characteristics of the given difference equation.*

4. A linear difference equation is to be considered as integrated when a domain of rationality is known for which the group of rationality is the identity, the elements $y_x^{(1)}, \cdots, y_x^{(\infty)}$ of a fundamental system being then rationally known. Thus the problem of integration consists in extending the domain of rationality until the group of rationality reduces to a subgroup. Consider the equation (8) with the rational function $\alpha([y_x], [y_{x+1}], \cdots)$ adjoined so that its group is G; adjoin now the rational function $\mathfrak{S}([y_x], [y_{x+1}], \cdots)$, which is invariant under the subgroup H of G but under no transformations of G not in H. It is evident that in this enlarged domain of rationality the group of the equation will be H.

In the above it was implicitly assumed that the group G is *continuous, i. e.,* is generated by infinitesimal transformations. Should G be a mixed (algebraic) group, then we denote its maximal invariant continuous subgroup by G_1, and by a well known theorem G consists of a certain number, say ν, sets of transformations

$$G_1,\ T_1G_1,\ T_2G_1,\ \cdots,\ T_{\nu-1}G_1,$$

where the T denote transformations for which

$$T_\lambda^{-1}G_1T_\lambda = G_1 \quad (\lambda = 1, 2, \cdots, \nu - 1).$$

If V denotes a characteristic invariant of G_1, *i. e.,* one which is invariant under the transformations of G_1 and no others, then by the adjunction of V the group of rationality reduces from G to G_1. Let the transformed of V through T_λ be V_λ ($\lambda = 1, 2, \cdots, \nu$); then the ν functions $V, V_1, V_2, \cdots, V_{\nu-1}$ satisfy an algebraic equation of the νth degree with rational coefficients and there results the theorem:

The group of rationality of a difference equation can be reduced to one which is continuous by the adjunction of the roots of an algebraic equation with rational coefficients.

5. A difference equation will be said to be *algebraically integrable* when its solutions satisfy a system of algebraic equations with rational coefficients.

Let the group G_1 of § 4 be the identical transformation, the group G becoming thus the finite group $T_0 = I, T_1, T_2, \cdots, T_{\nu-}$.

The linear difference equation is now algebraically integrable. For the characteristic invariants of G_1 being the elements of a fundamental system $[y_x]$, it follows that any rational function, and in particular any symmetric function of the $n\nu$ solutions $[y_x]$, $T_1[y_x]$, $T_2[y_x]$ $\cdots$, $T_{\nu-1}[y_x]$ remains invariant under the permutations of G and is therefore rational in x.

THE UNIVERSITY OF CHICAGO,
April, 1904.

EXPERIMENTAL AND THEORETICAL GEOMETRY.

Experimental and Theoretical Course of Geometry. By A. T. WARREN, M.A. Formerly Scholar of Corpus Christi College, Oxford. Assistant Master at Dover College. Oxford, at the Clarendon Press, 1903.

NOTWITHSTANDING the flood of books that have been put out in recent years under such titles as Experimental, Intuitional, Practical, Observational, Concrete, Heuristic and Objective Geometry, the common aim of which it has been to supply inductive knowledge of space relations, it is generally acknowledged by those who are practically engaged in the teaching of geometry that the ideal text-book is yet to be written. The tendency and danger in the class of books referred to has been to displace demonstrative geometry by offering in its place a pseudo-geometry — a conglomeration of interesting exercises well adapted to furnish the pupil with the facts of geometry, but imparting little if any training in close, consecutive thinking. In their attempt to escape the charges to which Euclid is open as a text-book for beginners, these books go to an opposite extreme and treat geometry as one would a natural science, forgetting that geometry proper is not an experimental science, that its essential object as a branch of study is not the discovery of facts but rather the discerning of relations between ideas. Indeed its very existence as an independent subject of study in the common and secondary schools is conditioned upon the recognition that its function differs from that of every other science, that whatever value may be placed upon its incidental results the one paramount virtue of geometry is that it develops the reasoning powers, just as the nat-

ural sciences develop the powers of observation and the judgment.

The book before us seems to be an attempt to avoid extremes. On the one hand it recognizes the need of a thorough grounding in the fundamental conceptions and facts and language of geometry, so that pupils may not fail for a want of adequate conceptions of the things reasoned about. On the other hand, it brings the experimental work within a compass that need not infringe seriously upon the time that should be given to geometry proper — demonstrative geometry.

The little volume contains 248 pages of open composition, of which 111 pages are devoted to experimental, the remaining 137 pages to theoretical geometry.

The experimental course consists of twenty-seven chapters. The general plan of each chapter is as follows:

1. The definitions of the principal terms are stated and carefully explained.

2. The essential properties and the relations of the various parts of the figure are developed inductively by means of the usual devices, most prominent among which are measurement of distances and angles by the use of linear scales and protractor, ruler and compass constructions, the use of tracing paper for transferring magnitudes, paper cutting and paper folding, rotation of lines and figures, the use of squared paper, construction of cardboard models.

3. A summary of conclusions which have been drawn.

4. A set of questions and exercises involving the concepts and relations that have been developed in the chapter.

To show the author's ingenuity and originality in the treatment of definitions we select two typical illustrations. Euclid's definition of a straight line is stated and explained as follows:

"*A* STRAIGHT *line is a line which lies* EVENLY *between its end points.*

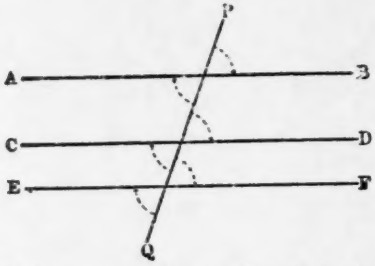
"To understand this definition, draw a line AB right across your paper. Cut the paper along AB; both edges of the paper, where it is cut, will be copies of the line AB. If now you can slide one edge in every possible way right along the other without leaving a space between them, the line AB is said to lie evenly between its end points A and B.

"Apply this test to show that the crease formed by folding a piece of paper is a straight line."

Chapter XVII opens with a definition of parallel lines:

"*Two straight lines are said to be* PARALLEL *when one can be moved* WITHOUT TURNING *so as to lie along the other.*

"Draw a straight line PQ to cut the ruled lines on your squared paper; let AB, CD and EF represent three of them. Over this lay a piece of transparent paper; on this take a tracing of PQ and of one of the straight lines met by PQ.



"Slide the transparent paper so that the tracing of PQ shall slide along PQ. The other traced line will thus move without turning, and it will be seen to coincide in turn with other ruled lines of the squared paper.

"You thus see that those ruled lines of the squared paper, which are not $\perp$ to one another are $\parallel$.

"It is also evident from the way in which certain $\angle$s coincide during the movement of the tracing, that the $\angle$s through which PQ would have to turn, in order to coincide with each of the $\parallel$ lines, are all equal."

It is further pointed out how the mechanical way of constructing parallel lines by sliding a set square along the edge of a T-square, satisfies the given definition of parallel lines.

The experimental methods employed in the discovery of relations are so various that it is difficult to select typical examples. In dealing with angles and angle sums, illustration by rotation of lines is most frequently employed. Thus by turning one of two intersecting lines about their point of intersection until it coincides with the other, it is shown that vertical angles are equal. By revolving one side produced of a triangle successively about the vertices of the triangle, through angles equal respectively to the corresponding angles of the triangle, the conclusion is drawn that the angle sum equals two right angles. Similarly it is shown that the exterior angle sum of

any polygon equals four right angles. Again, by counting the number of half revolutions which a line makes in revolving successively through consecutive angles of a polygon, the theorem concerning the interior angle sum of any polygon is derived.

The conversion of triangles and polygons into rectangles and of rectangles into squares and finally the quadrature of the circle is accomplished by paper cutting and the use of squared paper. Similar figures are defined as figures which are reduced or enlarged copies of one another, and their areas are compared by actually tabulating the number of equal elementary areas contained in figures having different linear dimensions.

An admirable feature of the experimental course is that the experiments can all be easily performed by the average pupil and that no apparatus and material is required that is not everywhere at hand.

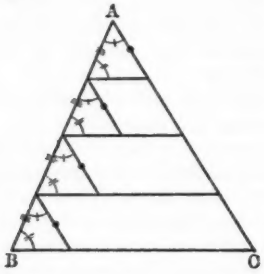
The exercises following each chapter could easily have been made more interesting and instructive. The sole aim seems to have been to give practice in the results already obtained, rather than to suggest new methods and applications. On the whole the impression is left that the exercises have not received the same care that has been bestowed on the body of the text.

Probably the greatest merit of the experimental course as compared with other texts of a similar character is its conciseness and brevity. This part of the work could be completed by a high school class, reciting daily, in ten to twelve weeks. It is excellently adapted as a brief introduction to demonstrative geometry for classes which have not had an adequate amount of concrete geometry in the grammar grades.

The theoretical course consists of 66 theorems and 17 problems of construction, 83 propositions in all. One is pleasantly surprised to find touches of originality in the treatment of a subject so thoroughly explored as theoretical geometry for beginners. Though we may not approve the manner of treatment in all particulars, one is forced to recognize that the work is that of a careful student and wide-awake teacher of geometry. Some of the most noticeable characteristics of the theoretical course are:

1. Elegance of proofs combined with simplicity in notation. The principle of continuity is tacitly assumed and frequently applied in dealing with limiting cases. Thus the theorem that the tangent to a circle is perpendicular to the radius drawn to the point of contact (Proposition 44) is derived by considering the

limiting value of the exterior angle of the isosceles triangle formed by a secant and the radii to its points of intersection with the circumference. Again, the angle formed by a tangent and a chord through the point of contact is treated as the limit of the exterior angle of the triangle formed by the chord and the two sides of an inscribed angle. Whenever possible, triangles, angles and lines are referred to by single letters or marks, an improvement of inestimable value over the cumbrous notation commonly employed. For instance, in the proof of the fundamental theorem on transversals (Proposition 51) the following figure is employed



whence the equality of the various parts is expressed by such simple statements as:

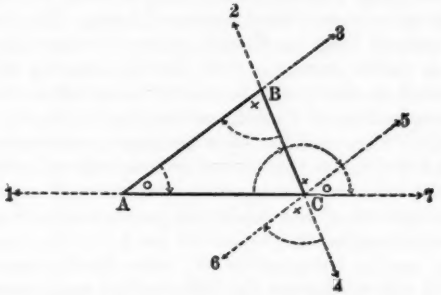
"All the ∠s marked / are equal; also all the ∠s marked // are equal; and the sides marked /// are equal; ∴ all the small △s are congruent, and so all their sides marked with a dot are equal; etc."

2. With two exceptions (Propositions 48 and 49) indirect proofs have been avoided. This is to be regretted, not only because in some cases (Euclid I, 25; I, 29; III, 21; III, 22.) the indirect proof excels all others in simplicity, but chiefly because of the intrinsic importance of the *reductio ad absurdum* principle with which the student should be made familiar as an instrument in exact reasoning.

3. Incommensurables are consistently ignored. Quantities of the same kind are treated as if they necessarily possessed some common measure. This procedure seems to us unwarrantable. Simplicity of treatment is purchased at too great a cost when it inculcates erroneous conceptions or does violence to truth itself. Better than to ignore intrinsic difficulties, is to

prepare for them by honorable recognition, though their treatment must of necessity be deferred to a later period if not indefinitely. It is easy to show even to beginners that the side of a square and its diagonal have no common measure, and thus to lay the foundation of a conception which permeates the whole of mathematics.

4. All axioms are introduced surreptitiously. One seeks in vain for explicitly assumed first principles. The angle sum of a triangle is proven equal to a straight angle by assuming that when the line lying along CA has been turned in succession about the points C, B and A until it again lies along CA the resultant turning is a half revolution. This of course is a fallacy.



Suppose that $C1$ has been turned about C clockwise until it lies along $C2$. Now while BC turns about B clockwise till it lies along BA, let $C4$ be turned about C through an equal angle and let its resulting position be along 56. The angles marked $\times$ are then equal. Let $A3$ now take its last turn about A until it lies along AC, and let $C5$ turn through an equal angle about C. We cannot say that $C5$ will take the direction of AC, without assuming the equality of the angles marked 0. This assumption justifies the conclusion that the sum of the turnings about C, B and A is equal to the straight angle $AC7$. But the assumption is no less than the theorem that two lines which make equal angles with one transversal make equal angles with every other transversal, the equivalent of Euclid I., 29.

The same theorem is smuggled into our author's definition of parallel lines. Parallel straight lines are defined as those which

can, without turning, be made to lie in the same straight line. Straight lines are defined as lines of which two cannot enclose a space. The phrase "can, without turning, be made to lie" means, as we gather from proposition 23, that they can be moved at a constant angle along a transversal. Along what transversal? Some specific one? Evidently not, for then the definition would be useless. But if along any transversal, the assumption is once more the equivalent of Euclid I., 29.

Finally the scope of the theoretical course is far too limited to commend itself as suited for American requirements. Moreover the selection of the propositions has not always been judicious. Limited exclusively to plane geometry and further to that small portion of it treated by Euclid, the course ignores many of the most important propositions which we are accustomed to find treated in American books. This weakness may be justified from an English point of view, though our author has parted company with Euclid in every other particular. But we also search in vain for some of the most important propositions of Euclid, such as Euclid II, 12; II, 13; IV, 10–15. The omission of a theorem so fundamental as that which expresses the square on one side of triangle in terms of the other two sides and the projection of one of these sides upon the other, can hardly be justified on any grounds. On the other hand we fail to see the need for the proposition on medial section (proposition 70), when by the omission of Euclid IV, 10, we deprive the student of its application. Still less seems there be a sufficient reason for introducing Ptolemy's theorem (proposition 60), which is of little other than historic value. As between Euclid II, 12, 13, and Euclid VI, B, the omission of the latter would have purchased brevity at the lesser cost.

We have given a more detailed review of this book than an English elementary text-book may seem to deserve, for two reasons. First, because of the unusual interest that is manifested at present in anything that aims at improvement in geometrical teaching, and second, because this little book bears a clear and distinct message to American teachers, a message that will be welcomed by thousands of teachers of geometry who are looking for light in the direction to which this book points and who will find it a valuable aid in their work in the class-room.

ROBERT E. MORITZ.

THE UNIVERSITY OF NEBRASKA,
April 15, 1904.

SHORTER NOTICES.

Elementary Calculus. By PERCY F. SMITH, Ph.D. New York, American Book Company, 1902, 1903. 8vo, 99 pp.

THE appearance of a new text book on elementary calculus is not a rare occurrence, but the first satisfactory answer to a definite question is always worthy of notice. In many technical schools students not in engineering courses are given an elementary course on the calculus, consisting of from thirty-five to fifty exercises. To make any "short course" successful is at best a difficult problem, and almost impossible of solution without a special text-book which can be completed in the alloted time. Otherwise the subject must be presented by lectures or by extracts from a larger text. Now a first or second year student wants the book, the whole book, and nothing but the book. Lectures on points not covered by the text are almost sure to be received with indifference, while whatever is in the book is accepted without question. If the course is given from a larger text the average student would be able to prove special theorems, solve problems, if he could guess under what case they occur, but would very probably lack any general idea of the subject.

In preparing a book to fill this want Professor Smith has written a true introduction to the fundamental ideas of the calculus. In case a rigorous proof would be beyond the scope of the book it is made clear that the explanation given is to illustrate the meaning of the theorem.

To the original five chapters of the book there has been added in the new edition another chapter of examples. The examples throughout are very well chosen, and the derivation of the derivatives and integrals of the elementary functions clearly given. The main interest centers in the introduction of the fundamental notions of the calculus — function, limiting value of a function, continuity, derivative and integral. In the definition of function and the exercises following, the idea is brought out that a function may be defined by an analytic law for some values of the independent variable and not for all; this point is usually omitted in elementary text books, and the omission is sadly felt when indeterminate forms are taken up.

After considering the graphs of some elementary functions, the idea of limit and limiting value of a function is introduced and a continuous function defined by the relation, limit $f(x) = f(\text{limit } x)$. In the theorems on limits, in preparation for derivative, the limit of $\frac{\sin\theta}{\theta}$ and of the series for e are taken up. No rigorous consideration of the limit of $\left[1+\frac{1}{z}\right]^z$ is given, but the quantity is calculated for ten values of z and the graph constructed as illustration of the fact that the limit is the previously defined quantity e. Exercises follow on the limiting values of some elementary functions.

Chapter II. gives a clear conception of derivative, the general definition being given after the consideration by text and examples of increment and increment quotient, and the special cases of velocity and slope of the tangent of a curve. General rules, the derivatives of the elementary functions and numerous examples follow.

Chapter III. takes up tangent and normal, maxima and minima, expansion of functions and differentials. Maxima and minima are treated very clearly. The theorem of the mean is stated, the proof being geometric. As a more general law of the same nature Taylor's expansion with remainder is given, but without proof. Differential of $f(x)$ is defined as the first term in the expansion of $f(x+\Delta x)-f(x)$. In the exercises the meaning of differential and infinitesimal of higher order is brought out by examples of area and volume.

Chapter IV. takes up the definition of integral, integrals of the elementary forms, definite integrals and limit of a sum. Chapter V. is devoted to functions of two or more variables, partial derivative, total differential and total derivative.

The press work on the book is good and the page — very different from the average English text — looks interesting. The book should meet with success, for by its aid the "short course" may become really worth while. MAX MASON.

Geometrische Transformationen. I Teil: Die projektiven Transformationen nebst ihren Anwendungen. Von DR. KARL DOEHLEMANN. Leipzig, Göschen, 1902. vii + 322 pp.

THE theory of transformations has come to be of fundamental importance in geometry and yet the number of books devoted

to a systematic treatment of the theory is very limited. Under the title of geometric transformations, Doehlemann proposes to develop the properties of those transformations which are susceptible of a purely geometric treatment. This line of demarcation is, as the author recognizes, not very definite; but it excludes, for example, the consideration of the totality of conformal transformations in the plane. The volume at hand is devoted to projective transformations and will be followed by a second volume dealing with Cremona transformations and higher correspondences.

The volume is divided into four parts dealing with projective coördinates (pages 1–64) and the projective transformations on the line (pages 65–105) in the plane (pages 106–247) and in space (pages 248–320). The geometric point of view is kept in the foreground, as the avowed object of the book is to increase the pleasure of geometric studies, although analytic methods are used freely. Many applications are considered and, in particular, considerable attention is devoted to the applications to descriptive geometry.

The chapters on projective coördinates require but little comment. Various properties of cross ratios are derived and then the projective coördinates of points of a range and lines of a pencil are defined by the use of the cross ratio. The parametric representation is considered and the angle between two lines expressed as a cross ratio. The treatment of coördinates of points and lines in a plane is excellent and the discussion of the coördinates in space is as detailed as that for the plane.

A projective transformation of a given fundamental configuration into a second of the same number of dimensions, as the points of a plane into the lines of a second plane, is defined by putting those elements in correspondence whose coördinates are proportional. Each of the last three parts of the book opens with a discussion of the transformation thus defined, which leads to the invariance of the cross ratio and the number of pairs of corresponding elements necessary to determine the transformation. Then comes the discussion of the general linear transformation and some attention is paid to the degenerate cases in which the determinant of the transformation is zero. In each of these three parts the transformation of one fundamental configuration into a second is completely discussed before the transformation of a configuration into itself is considered.

The parametric representation of a one-dimensional configuration is developed at length and much attention is paid to cyclic projectivities and, in particular, to involution. In the plane and in space the Möbius net is used considerably and correlations as well as collineations receive their due share of attention.

In dealing with the transformation of a line, a plane or of space into itself the nature of the invariant configuration is not developed with as much completeness as it might have been, nor are the canonical forms of the equations of the transformations as thus classified given even for the case of the line.

The group idea is not considered at all although the word group is used twice, parenthetically. It would seem reasonable that, in a book of over three hundred pages devoted to the consideration of projective transformations only, space might be found to show at least the relations which the euclidean and projective geometries bear to their respective groups.

The appearance of the book is sufficiently described by saying that it appears in the Sammlung Schubert. There are some typographical errors, of which those noticed as most apt to confuse the student may be mentioned here. On page 8 in the definitions of h_1 and k_2 the e_1 and e_2 should be interchanged; on page 71, line 14, for 5 read 7; on page 76, line 1, for 40 read 11; and on page 87, line 15, b_1' and b_2' should be interchanged. An error appears on page 92: the third line from the bottom should read

$$c = 0,\ a + b = 0 \text{ and } b + d = 0,$$

so that the last equation on the page should be

$$\lambda\lambda' - \lambda + 1 = 0.$$

It is then unnecessary to make the change in parameters on page 93 which shows the identity of the two cyclic projectivities under discussion.

The book contains much that is interesting and helps to fill a decided gap in the works on geometry.

ARTHUR SULLIVAN GALE.

Aufgaben aus der niederen Geometrie. Von IWAN ALEXANDROFF, mit einem Vorwort von Dr. M. SCHUSTER. Leipzig, Teubner, 1903. vi + 123 pp.

ANYTHING tending to systematize the solution of the "originals" of elementary geometry is welcomed as an aid to

the study of geometry, and the value of Alexandroff's book is evidenced by the translations which have appeared in France and now in Germany. The book consists of approximately a thousand problems of construction, many of which are solved.

The first chapter of 10 pages consists of simple constructions and theorems of a fundamental nature.

The second chapter dealing with problems of construction and methods of solution covers pages 11–91 and forms the most important part of the book. It is noteworthy for its systematic development. The methods of construction employed are eight in number; the method of geometric loci, transformations of similitude, solving the converse problem, transformations of symmetry, translation, rotation about an axis, rotations about a point, and inversion.

In a third chapter, pages 92–108, problems from plane and solid geometry are solved by the use of algebra, and in the concluding chapter miscellaneous problems are found.

While the book is too advanced for use in the preparatory schools, it might well be placed in the hands of those students who plan to teach in these schools without having done graduate work.

ARTHUR SULLIVAN GALE.

Conceptos fundamentales de Analysis matematica. Por LAURO CLARIANA RICART, Catedratico de la Universidad de Barcelona. Barcelona, Juan Gili, 1903. 192 pp.

THE learned author states in his preface that the aim of the book is to contribute to the popularization of mathematical science by treating not of the subjects usually contained in elementary manuals, but of those fundamental concepts of mathematical science which are of special importance either for their historical value or for their utility in science. The contents embrace negative quantities, imaginary quantities, the equipollences of Bellavitis, the quaternions of Hamilton, circular and hyperbolic functions, derived functions, congruences, the symbols of Wronski, the theory of algebraic forms, etc. The author treats not only of the fundamental ideas of these branches of analysis, but also of their fundamental principles. His presentation is marked by scientific exactness and philosophical penetration. To one who can read Spanish and who has been truly initiated into mathematical science, the book must be charming, for it serves as a guide to several of the most interesting fields of exact science.

ALEXANDER MACFARLANE.

Carl Anton Bjerknes: Gedächtnisrede. Von V. BJERKNES. Leipzig, J. A. Barth, 1904. 31 pp. + photo.

To some it may appear strange that the son of C. A. Bjerknes should have been chosen to deliver the commemorative address summing up the life-work of his father. As a matter of fact, however, no other choice could have been made. In his scientific research Bjerknes worked apart from others. His only confidant and colleague was his son. So in the monograph before us the son, after sketching his father's early life, traces step by step the development of the Hydrodynamic Action at a Distance from the days when its author was a pupil under Cauchy, Lamé, and Dirichlet until the last manuscript, written two or three days before his sudden death by apoplexy. Bjerknes left about 40,000 pages of closely written manuscript, accumulated since the early seventies. So great was his love of perfection, his striving for quality rather than quantity that little of all this had been published until the appearance of the Hydrodynamische Fernkräfte* and that which remains is accompanied by a request that nothing be printed without the most careful revision. It is a rare and noble sight to see men like Gibbs and Bjerknes who are possessed of a spirit of research apart from the common desire to rush into print.

E. B. WILSON.

Tablas de Multiplicar. By J. DE MENDIZÁBAL Y TAMBORREL. Mexico, Imprenta y eucuadernacion de Mariano Nava, 1903.

SOME years ago the author of this volume published a set of 8-figure logarithm tables of the numbers up to 120,000. Curiously enough, similar tables were published by the French government in the same year, these two being the first complete 8-figure tables to appear. M. Tamborrel now makes a decided advance in one direction on Crelle's multiplication tables which gave the products of all numbers up to 1000 by numbers up to 1000. He has arranged in a handy form the products of all numbers from 1 to 10,000 by numbers from 1 to 100. To save space in printing, these are arranged in a novel manner. Down the centre column of each page run the numbers from 1 to 99. On the right in nine columns are shown the products of these by nine 3-figure numbers; on the left, also in nine col-

* Reviewed in the BULLETIN, December, 1903.

umns, are shown the figures that must be substituted for the thousands, etc., when another digit is placed *in front* of the 3-figure number. Thus 63×137 is shown directly on the right to be 8631; the product of 63×5137 is obtained by extracting from the corresponding line on the left the number 323 which is to be substituted for the 8 of the previous product, thus giving 323,631. When an extra unit has to be added to the thousands, the last three figures in the product on the right are printed in smaller type with a bar placed over them. In this way it is possible to get a product of a 4-figure number by a 3- or 4-figure number with only one addition. Examples are added to show how longer multiplications, and also how divisions by 3-figure numbers are to be performed. These tables occupy $100 - 10 = 90$ pages, in each of which the printed matter takes up a space of $10\frac{1}{2}$ by $5\frac{1}{2}$ inches. In five additional pages of the same size and arranged in a similar manner are given the squares of all numbers up to 10,000 and the cubes to 1,000. For concentration of space combined with moderate convenience of use, it would be difficult to improve on these results. It would perhaps have been better, even at the cost of a somewhat longer page, to separate the lines a little more, as one has to exercise rather too much care in running the finger along a line.

Accompanying the tables is a slip containing 16 errata which from their nature appear to be mainly typographical. It is to be hoped that there are no others outstanding: no indication is given as to the means adopted to secure accuracy. The author is to be congratulated on the completion of a task which must have been exceedingly tedious and uninteresting, but which will undoubtedly be highly appreciated by those who have much to do with calculations in which these tables can be of service.

ERNEST W. BROWN.

NOTES.

THE eleventh summer meeting of the AMERICAN MATHEMATICAL SOCIETY will be held in the hall of congresses, on the exposition grounds in St. Louis, Missouri, on Friday and Saturday, September 16 and 17. The date is so fixed as to enable members conveniently to attend the international congress of arts and science, instituted in connection with the Louisiana Purchase Exposition. This congress will take place during the week following the meeting of the Society, mathematics (department 2) being assigned to Tuesday, Thursday and Saturday, September 20, 22, 24. (For list of speakers see previous announcement in June BULLETIN, page 471).

Titles and abstracts of papers intended for presentation at the summer meeting of the Society should be sent to the Secretary at an early date for submission to the programme committee. A supplementary account of each paper would be helpful to the committee of arrangements for the meeting, and should be enclosed with the abstract to the Secretary. Such accounts should state briefly the history of the topic discussed, its connection with other papers, particularly those of the same author, and should indicate one or two references that will aid a reader to understand the paper itself more easily. The committee will use this information in an attempt to insure more thorough and interesting discussion of papers read at the meeting. For the same reason, those members who expect to attend are requested to report that intention to the Secretary by August 25. A list of titles of papers received before August 25 will be printed and mailed to all members of the Society.

The headquarters of the Society will be at the Inside Inn, on the exposition grounds. The members of the committee in charge of the summer meeting are Professors H. S. WHITE, A. S. CHESSIN, and E. R. HEDRICK.

THE April number (volume 26, number 2) of the *American Journal of Mathematics* contains: "On null-systems in space of five dimensions and their relations to ordinary space," by J. EIESLAND; "On the forms of unicursal quintic curves," by P. FIELD; "Determination of the algebraic curves whose polar conics are parabolas," by E. KASNER; "On certain conics con-

nected with trinodal quartics," by A. B. BASSET; "A geometric proposition," by E. LASKER; "Congruences of tangents to a surface and derived congruences," by L. P. EISENHART.

AT the meeting of the London mathematical society held on May 12, the following papers were read: By Mr. C. COOKE, "Some mathematical instruments"; by Mr. A. L. DIXON, "Evaluation of certain definite integrals by means of gamma functions"; by Mr. A. L. DIXON, "Generalization of Legendre's formula $KE' - (K - E)K' = \pi/2$"; by Dr. H. F. BAKER, "Note on the integration of linear differential equations"; by Mr. A. YOUNG and Mr. P. W. WOOD, "Perpetuant syzygies"; by the Rev. F. H. JACKSON, "Some properties of the gamma function"; by Mr. G. B. MATTHEWS, "On the geometric representation of imaginaries."

At the meeting held on June 9, the following papers were read: By LORD RAYLEIGH, "The application of Poisson's formula to discontinuous disturbances"; by Mr. H. BATEMAN, "Some expansions for the periods of the Jacobian elliptic functions"; by Mr. P. W. WOOD, "Type of covariants of any degree in the coefficients of each of any number of binary quantics."

THE programme of the St. Louis meeting of the National educational association included a mathematical conference, held on June 28. In connection with this conference arrangements were made for a meeting of teachers of mathematics in Missouri schools and colleges, for the purpose of promoting improved professional relations.

THE house of B. G. Teubner in Leipzig announces the early publication of: Felix Müller, Einführung in die mathematische Literatur. The book, which contains about 200 pages, is not intended to give a complete mathematical bibliography, but rather to guide students into the literature of the subject in the spirit of the recent plans of study issued by various German universities.

THE universities below announce the following mathematical courses during the winter semester of 1904–1905:

UNIVERSITY OF GÖTTINGEN. — By Professor F. Klein: Pedagogy of mathematics I, four hours; Seminar, with Professor Schwarzschild, two hours. — By Professor D. HILBERT:

Calculus of variations, four hours; Seminar on mechanics, with Professor MINKOWSKI, two hours. — By Professor H. MINKOWSKI: Differential and integral calculus II, four hours; with exercises, two hours; Analysis situs, two hours. — By Professor M. BRENDEL: Calculus of probabilities, two hours; Seminar on insurance, two hours. — By Professor W. VOIGT: Elements of vector analysis and mechanics, four hours; Theory of potential, two hours; Seminar in vector analysis, two hours. — By Professor F. SCHILLING: Graphical statics and kinematics, four hours; with exercises, two hours. — By Dr. E. ZERMELO: Theory of algebraic equations, four hours. By Dr. O. BLUMENTHAL: Developments of series in physics, four hours. — By Dr. BOSE: Higher mathematics for students of science, four hours.

UNIVERSITY OF PARIS (beginning November 7, 1904). — By Professor G. DARBOUX: General principles of infinitesimal geometry, two hours. — By Professor E. GOURSAT: Applications of differential and integral calculus, two hours. — By Professor P. PAINLEVÉ: General laws of motion and of equilibrium, two hours. — By Professor P. APPELL: Elements of mathematics preparatory to the study of mechanics, three hours. — By Professor H. POINCARÉ: Determination of planetary orbits, two hours. — By Professor J. BOUSSINESQ: Theory of elasticity, two hours. — By Professor G. KOENIGS: General kinematics, two hours. Mathematical conferences will be held as follows: By Professors F. RAFFY and J. HADAMARD: Calculus. — By Professor P. PUISEUX: Theory of newtonian potential.—By Professor J. HADAMARD and Mr. E. M. BLUTEL: General conferences for students of mathematics.

THE various American universities below announce the following advanced mathematical courses during the year 1904–1905:

UNIVERSITY OF COLORADO. — By Professor I. M. DELONG: Any two of the following courses: Theory of equations, five hours; Advanced calculus, five hours; Theory of plane curves, five hours; Theory of investments, three hours. — By Professor A. EMCH: Any two of the following courses: Chapters in higher applied mathematics, three hours; Elliptic functions, three hours; Functions of a complex variable, five hours; Projective geometry, five hours.

UNIVERSITY OF ILLINOIS. — By Professor S. W. SHATTUCK: Differential equations and calculus of variations, three hours. — By Professor A. N. TALBOT: Analytic mechanics, four hours. — By Professor E. J. TOWNSEND: Theory of functions, three hours; Seminar, three hours; Partial differential equations, first semester, three hours; Analytic geometry of space, second semester, three hours. — By Professor A. G. HALL: Potential function and spherical harmonics, three hours; Determinants, two hours. — By Dr. H. L. COAR: Modern geometry, first semester, three hours; Algebraic surfaces, second semester, three hours. — By Professor H. L. RIETZ: Theory of invariants and higher plane curves, three hours. — By Dr. J. STEBBINS: Least squares, first semester, two hours. — By Mr. E. L. MILNE; Mathematical theory of statistics, first semester, four hours.

INDIANA UNIVERSITY. — By Professor R. J. ALEY: Differential equations, two hours; Modern geometry, three hours, fall term; Algebra of quantics, two hours, fall and winter terms; Theory of numbers, three hours, spring term. — By Professor S. C. DAVISSON: Modern analytic geometry and higher plane curves, two hours, winter and spring terms; Fourier series, two hours, fall and winter terms; Theory of surfaces, two hours, winter and spring terms. — By Professor D. A. ROTHROCK: Advanced calculus, three hours; Projective geometry, two hours, fall and winter terms; Theory of continuous groups, two hours, winter and spring terms. — By Professor U. S. HANNA: Advanced conics, two hours, winter and spring terms; Groups of substitutions and Galois theory, three hours, winter and spring terms.

STATE UNIVERSITY OF IOWA. — By Professor L. G. WELD: Determinants and modern geometry, two hours; Definite integrals, first semester, two hours; Elliptic functions, second semester, two hours; Theory of potential, trigonometric series and spherical harmonics, three hours. — By Professor A. G. SMITH: Method of least squares, two hours; Theoretical mechanics, two hours. — By Dr. J. V. WESTFALL: Advanced calculus, introducing to higher analysis, three hours; Theory of surfaces, second semester, two hours; Differential equations, advanced course, three hours. The mathematical seminary meets one evening each week.

JOHNS HOPKINS UNIVERSITY. — By Professor F. MORLEY: Higher geometry, two hours; Theory of groups, first semester, two hours; Vector analysis, second semester, two hours; Classic authors, one hour. — By Dr. A. COHEN: Elementary theory of functions, two hours; Ordinary differential equations, two hours; Rational mechanics, first semester, two hours; Calculus of variations, second semester, two hours. — By Dr. A. B. COBLE: Theory of invariants, two hours. — By Dr. F. FRANKLIN: Theory of probability, second semester, two hours.

UNIVERSITY OF MISSOURI. — By Professor E. R. HEDRICK: Theory of functions, three hours; Differential equations, three hours; Advanced calculus, three hours. — By Professor L. M. DEVOE: Fourier series and potential function, three hours; Analytic mechanics, three hours. — By Professor G. A. BLISS: Advanced analytic geometry, three hours; Theory of groups, three hours. — By Mr. L. D. AMES: Galois theory of substitutions, three hours; Infinite series and products, three hours. — By Mr. INGOLD: Projective geometry, three hours; Theory of equations, three hours.

PROFESSOR P. GORDAN has been elected corresponding member of the academy of sciences of Paris, section of geometry, as successor of the late Professor G. Salmon, and also corresponding member of the academy of sciences of Vienna.

PROFESSOR G. DARBOUX has been elected member of the Bureau des longitudes of France.

PROFESSOR SIMON NEWCOMB has been elected corresponding member of the Berlin academy of sciences.

PROFESSOR A. KNESER has declined a call to a professorship of mathematics at the University of Marburg. Dr. O. BLUMENTHAL of Göttingen is giving a course of lectures at Marburg during the present semester.

Dr. G. HESSENBERG has been promoted to an assistant professorship of mathematics at the technical high school at Charlottenburg.

PROFESSOR G. LANDSBERG, of the University of Heidelberg, has been appointed professor of mathematics at the University of Strassburg.

PROFESSOR M. COLLET has been appointed dean of the faculty of sciences at the University of Grenoble.

PROFESSOR N. HATZIDAKIS, of the Higher military school of Greece, has been appointed to a full professorship of analysis at the University at Athens.

PROFESSOR J. M. VAN VLECK, of Wesleyan University, has been made professor emeritus.

AT Cornell University, assistant professors JAMES MCMAHON and J. H. TANNER have been promoted to full professorships of mathematics.

MR. A. S. EVE has been appointed lecturer in mathematics at McGill University.

DR. C. L. E. MOORE has been appointed instructor in mathematics at the Massachusetts Institute of Technology.

AT the University of Illinois, DR. H. L. RIETZ has been promoted to an assistant professorship of mathematics; Mr. E. B. LYTLE and Mr. F. C. TOUTON have been appointed instructors in mathematics.

AT the State University of Iowa, the department of physics has been divided and materially enlarged. Professor A. A. VEBLEN will continue to act as professor of experimental physics; while Professor A. G. SMITH, formerly of the chair of mechanics in the department of mathematics, assumes the title of professor of physics and mechanics. Dr. R. T. WELLS will retain the rank of professor in charge of electrical and mechanical engineering. An instructorship in mathematics remains to be filled; as also a professorship in civil engineering.

MISS MARY ANDERSON has been appointed professor of mathematics in the Illinois Women's College, Jacksonville, Illinois.

DR. A. B. COBLE, of the University of Missouri, has been appointed instructor in mathematics at Johns Hopkins University.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ABEL (N. H.). Ein Brief an Edmund Jacob Külp, herausgegeben von C. Störmer. Christiania, Dybwand, 1903. 8vo. 8 pp. M. 0.50

CAMPAGNE (M.). De l'emploi des chiffres dits arabes au moyen âge. 1904. 8vo. 42 pp. 4 plates.

CURTISS (D. R.). Binary families in a triply connected region, with especial reference to hypergeometric families. (*Memoirs of the American Academy of Arts and Sciences*, Vol. 13, No. 1.) Cambridge, Wilson, 1904. 4to. 59 pp.

DICKSTEIN (S.). See KOCHANSKI and PASCAL (E.).

DUCCI (E.). Le mie Lezioni di analisi indeterminata di primo grado nel r. Istituto tecnico di Melfi, anno 1902–03. Bologna, 1903. 8vo. 30 pp.

ERNST (G.). De geometricis illis quae sub Boëthii nomine nobis tradita sunt quaestiones. Bayreuth, 1904. 8vo. 32 pp. M. 1.50

FORBES (C. S.). The geometry of circles orthogonal to a given sphere. (Diss., Columbia University.) New York, 1904. 4to. 4 + 50 pp.

GEER (P. VAN). Analytische meetkunde. Oplossingen der vraagstukken, von H. Siersma. Culemborg, 1902. 8vo. 5 + 224 pp. M. 4.50

GRÜTTNER (A.). Das räumliche Fünfeck. (Diss.) Breslau, 1903. 8vo. 85 pp.

HINTON (C. H.). The fourth dimension. London, Sonnenschein, 1904. 8vo. 256 pp. Cloth. 4s. 6d.

HUMBERT (G.). Cours d'analyse, professé à l'Ecole polytechnique par G. Humbert. Vol. 2: Compléments du calcul intégral (fonctions analytiques et elliptiques; equations différentielles). Paris, Gauthier-Villars, 1904. 8vo. 18 + 496 pp. Fr. 16.00

KEYSER (C. J.). The axiom of infinity; a new presupposition of thought. 8vo. (*The Hibbert Journal*, 2 (1904), pp. 532–552.)

KOCHAŃSKI i LEIBNIZ. Korrespondencya Kochańskiego i Leibniza wedlug opisów L. Bodemanna, wydana przez S. Dicksteina. Warszawa, 1902. 8vo. 85 pp. M. 1.80

LARKMAN (A. E.). The calculus for engineers and others. Specially adapted for board of trade examinations. London, Simpkin, 1904. 8vo. 92 pp. Cloth. 4s. 6d.

LAURENT (H.). Théorie des nombres ordinaires et algébriques. Paris, Naud, 1904. 8vo. 2 + 185 pp.

LEIBNIZ. See KOCHAŃSKI.

LOPUSZAŃSKI (T.). Z podstaw teoryi funkcyj. Kraków 1903. 8vo. 113 pp.

LOSERT (B.). Das Kegelschnittbüschel. Wien, 1903. 8vo. 37 pp.

MACFARLANE (A.). Bibliography of quaternions and allied systems of mathematics. Dublin, University Press, 1904. 8vo. 86 pp.

MORAND (L.). Généalogie de la famille de Gaspard Monge. Paris, Nourry, 1904. 16mo. 52 pp. and portraits.

MUIR (T.). A third list of writings on determinants. (*Report of the South African Association for the Advancement of Science*, Vol. 1, April, 1903.) 4to. Pp. 154–228.

ORLANDO (L.). Esercizi di calcolo infinitesimale. Vol. 1. Messina, 1903. 12mo. 119 pp. Fr. 3.00

PASCAL (E.). Résumé de quelques-uns de mes récents travaux sur la théorie des groupes de Lie. Varsovie, 1903. 8vo. M. 1.00

——. Grupy ciagle przeksztalcien. Cześć ogólna teoryi. Przelozyl S. Dickstein. Warszawa, 1903. 8vo. 12 + 296 pp. M. 7.50

RUNGE (C.). Theorie und Praxis der Reihen. Leipzig, Göschen, 1904. 12mo. 266 pp. Cloth. (Sammlung Schubert, Vol. XXXII.) M. 7.00

SCHIFF (W.). Collection of exercises and problems in plane and solid analytic geometry. St. Petersburg, 1904. 4to. 122 pp. (Russian.) M. 4.00

SCHLESINGER (L.). Einführung in die Theorie der Differentialgleichungen mit einer unabhängigen Variabeln. 2te, revidierte Auflage. Leipzig, Göschen, 1904. 12mo. 320 pp. Cloth. (Sammlung Schubert, Vol. XIII.) M. 8.00

SIERSMA (H.). See GEER (P. VAN).

STÖRMER (C.). See ABEL (N. H.).

TORROJO Y CABALLÉ (E.). Teoría geométrica de las líneas alabeadas y de las superficies desarrolladas. Madrid, Fortanet, 1904. 8vo. 388 pp. Fr. 15.00

VOIGT (H.). Eléments de mathématiques supérieures, à l'usage des physiciens, chimistes et ingénieurs et des élèves des facultés des sciences. 2e édition. Paris, 1904. 8vo. 7 + 619 pp.

WALLSTAFF (W.). Ueber eine besondere Cremona'sche Transformation. Breslau, 1902. 8vo. 72 pp. M. 2.20

WESTPHAL (W.). Uebertragung einer Dreiecksaufgabe auf das Tetraeder. (Diss.) Marburg, 1904. 8vo. 65 pp.

II. ELEMENTARY MATHEMATICS.

BACKHAUS (K.). See WIESE (B.).

BARDEY (E.). Aufgabensammlung, methodisch geordnet, mehr als 8000 Aufgaben enthaltend, über alle Teile der Elementar-Arithmetik, vorzugsweise für Gymnasien, Realgymnasien und Oberrealschulen, sowie für Seminare und Präparanden-Anstalten. Neue Ausgabe, bearbeitet von F. Pietzker und O. Presler. 3te Auflage. Leipzig, Teubner, 1904. 8vo. 8 + 395 pp. Cloth. M. 3.20

BOREL (E.). Trigonométrie (second cycle). Paris, Colin, 1904. 16mo. 8 + 198 pp. Fr. 2.50

BÖTTCHER (R.) und SENDLER (R.). Raumlehre für Lehrerseminare. Nach dem Lehrplan vom 1. VII. 1901 bearbeitet. Teil 2: Stereometrie und Trigonometrie. Breslau, Handel, 1904. 8vo. 4 + 105 pp. M. 1.00

CAMOSCI (A.). L'aritmetica intuitiva e la geometria sperimentale per le classi superiori delle scuole elementari. Torino, Paravia, 1904. 16mo. 144 pp. Fr. 0.90

CEHAK (A.). Istota znaków matematycznych. Lwów, 1903. 4to. 25 pp.

CORREA (J.). Elementos de algebra. Zaragoza, 1903. 8vo. 251 pp.

DEAKIN (R.). Euclid, books 1 and 2. King's scholarship edition. With questions set at scholarship examinations during last 8 years. London, Clive, 1904. 8vo. 196 pp. Cloth. 1s.

EUCLID. See DEAKIN (R.).

FRANZ (R.). See UTH (K.).

FUSS (K.). Sammlung von Aufgaben aus der Buchstabenrechnung und Algebra, für Schulen und zum Selbstunterricht. 6te, verbesserte Auflage. Nürnberg, Korn, 1904. 8vo. 12 + 256 pp. M. 2.40

HAMILTON (J. G.) and KETTLE (F.). Graphs and imaginaries. An easy method of finding graphically imaginary roots of quadratic equations and imaginary points of intersections of various curves, with illustrations of the principle from elementary geometry. London and New York, Longmans, 1904. 12mo. 6 + 41 pp. Cloth. $0.50

HULL (G. W.). Elements of algebra for beginners. New York, American Book Co., 1904. 12mo. 2 + 159 pp. Cloth. $0.50

JANISCH (W.). Geometrische Aufgaben zur Lehre von der Proportionalität der Grössen (Streckenteilung, vierte und mittlere Proportionale, Aehnlichkeit der Figuren, Strecken am Kreise, stetige Teilung). Potsdam, Stein, 1904. 8vo. 100 pp. M. 1.50

KETTLE (F.). See HAMILTON (J. G.).

KÖLTZSCH (A.). Raumlehre für Präparanden. Nach dem "Lehrplan für die Präparandenanstalten" des k. preussischen Ministers der geistlichen, Unterrichts- und Medizinal-Angelegenheiten (Erlass vom 1. VII. 1901) bearbeitet. Mit etwa 800 Uebungsaufgaben. Leipzig, Merseburger, 1904. 8vo. 128 pp. Boards. M. 1.20

LAUNAY (L.). Calcul et géométrie intuitive. Classes élémentaires, classes de huitième et septième. 8e edition. Paris, Delagrave, 1903. 18mo. 251 pp.

LICHTBLAU (W.). See WIESE (B.).

LIGHTFOOT (J.). Studies in graphic arithmetic. (Normal Tutorial Series.) London, Simpkin, 1904. 8vo. Limp leather. 1s. 6d.

MARTINAUD (A.). Arithmétique théorique et géométrie (premier cycle), à l'usage de l'enseignement secondaire, rédigées conformément aux programmes officiels de 1902. Paris, Belin, 1904. 12mo. 8 + 280 pp.

PIETZKER (F.). See BARDEY (E.).

PONS Y MERI (B.). Elementos de trigonometría rectilínea. Valladolid, 1904. 8vo. 78 pp.

PRESLER (O.). See BARDEY (E.).

SCHUSTER (M.). Geometrische Aufgaben und Lehrbuch der Geometrie. Planimetrie, Stereometrie, ebene und sphärische Trigonometrie. Nach konstruktiv-analytischer Methode bearbeitet. Ausgabe A: Für Vollanstalten. Teil I: Planimetrie. 2te, nach den preussischen Lehrplänen von 1901 umgearbeitete Auflage. Leipzig, Teubner, 1903. 8vo. 10 + 154 pp., 2 plates. Cloth. M. 2.00

SENDLER (R.). See BÖTTCHER (R.).

SHAW (W. N.). First lessons in observational geometry. London and New York, Longmans, 1904. 12mo. 10 + 148 pp. Cloth. $0.60

STEUER (W.). Raumlehre für Volksschulen, enthaltend die Ergebnisse des Unterrichts und Aufgaben. Breslau, Woywod, 1904. 8vo. 3 + 52 pp. Boards. M. 0.45

TANNER (J. H.). Elementary algebra. New York, American Book Co., 1904. 8vo. 10 + 364 pp. Half leather. $1.00

UTH (K.). Planimetrie. Leitfaden mit Konstruktionsaufgaben und Uebungssätzen. 7te Auflage, 2te der Neubearbeitung von R. Franz. Kassel, Hühn, 1904. 8vo. 8 + 157 pp. Cloth. M. 2.00

WIESE (B.), LICHTBLAU (W.) und BACKHAUS (K.). Raumlehre für Lehrerbildungsanstalten. (In 2 Teilen.) Teil I: Planimetrie (Flächenlehre). Abteilung 1: Der Lehrstoff für die Präparandenanstalt. 150 pp. Abteilung 2: Der Lehrstoff für das Seminar. 106 pp. 6te Auflage, umgearbeitet und erweitert nach dem Lehrplan vom 1. VII. 1901. Breslau, Hirt, 1904. 8vo. Boards. M. 3.00

III. APPLIED MATHEMATICS.

ARNOLD (E.). La machine dynamo à courant continu (théorie, construction, calcul, essais et fonctionnement). Traduction française par E. Boistel. Vol. 1: Théorie de la machine à courant continu. Paris, Béranger, 1904. 8vo. 21 + 627 pp.

ATTWOOD (E. L.). The text-book of theoretical naval architecture. London and New York, Longmans, 1904. 12mo. Cloth. $2.50

BOISTEL (E.). See ARNOLD (E.).

BÜRGI (R. T.). Der Elektronäther. Beiträge zu einer neuen Theorie der Elektrizität und Chemie. Berlin, 1904. 8vo. 47 pp. M. 1.20

CALDARERA (F.). Corso di meccanica razionale. Vol. II: Statica; dinamica. Fascicolo 2. Palermo, Tipografia matematica, 1903. 8vo.

CANEVAZZI (S.). Meccanica applicata alle costruzioni. Testo e tavole. Torino, Negro, 1904. 8vo.

COMBETTE (E.) et GIROD (J.). Leçons de mécanique, conformes aux programmes du 31 mai 1902, pour les classes de première C et D et de mathématiques A et B. Paris, Alcan, 1904. 8vo. 329 pp. Fr. 3.50

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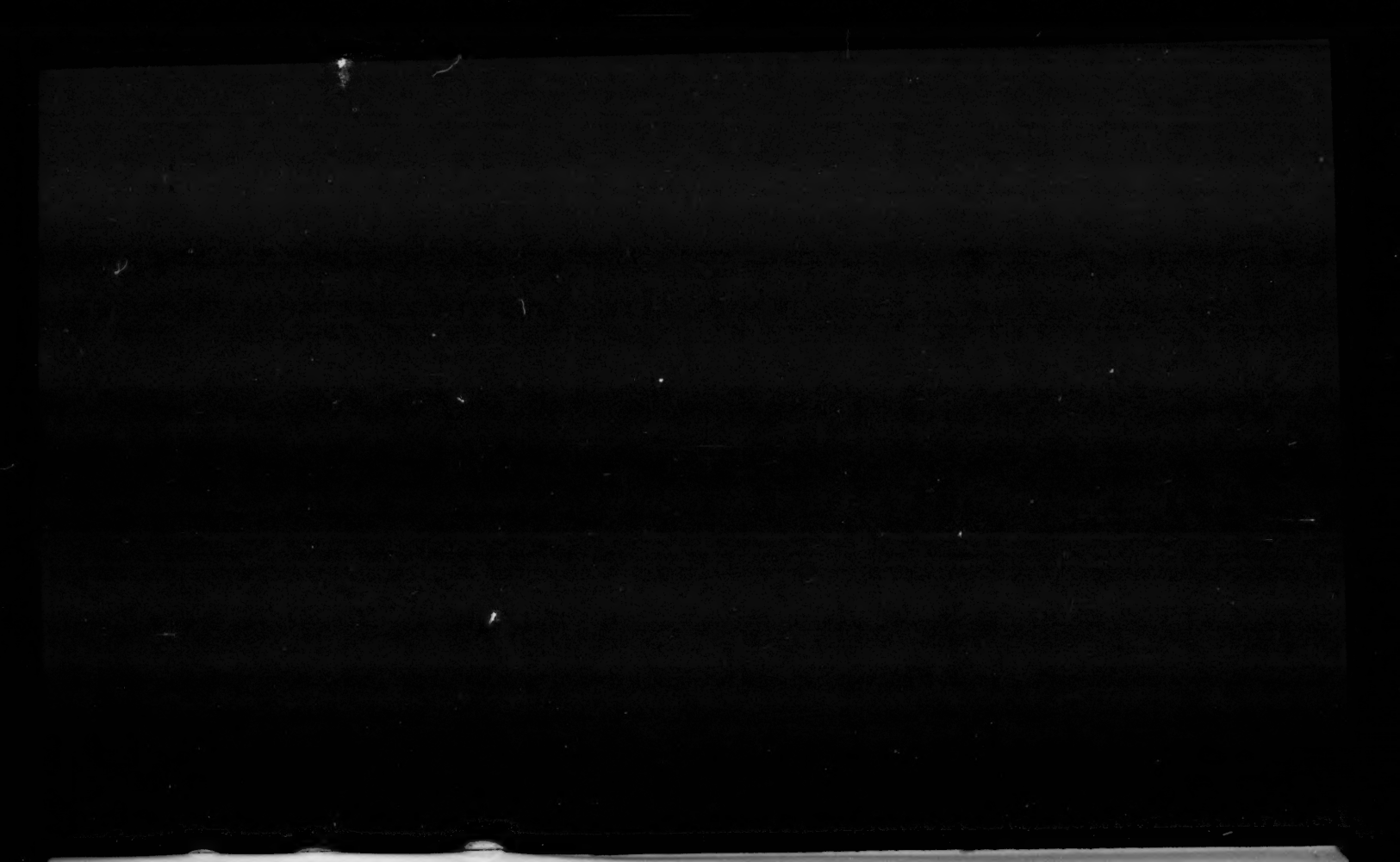
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